NUMBERS & OPERATIONS 01-01

Furen International School
O Level Foundation Mathematics

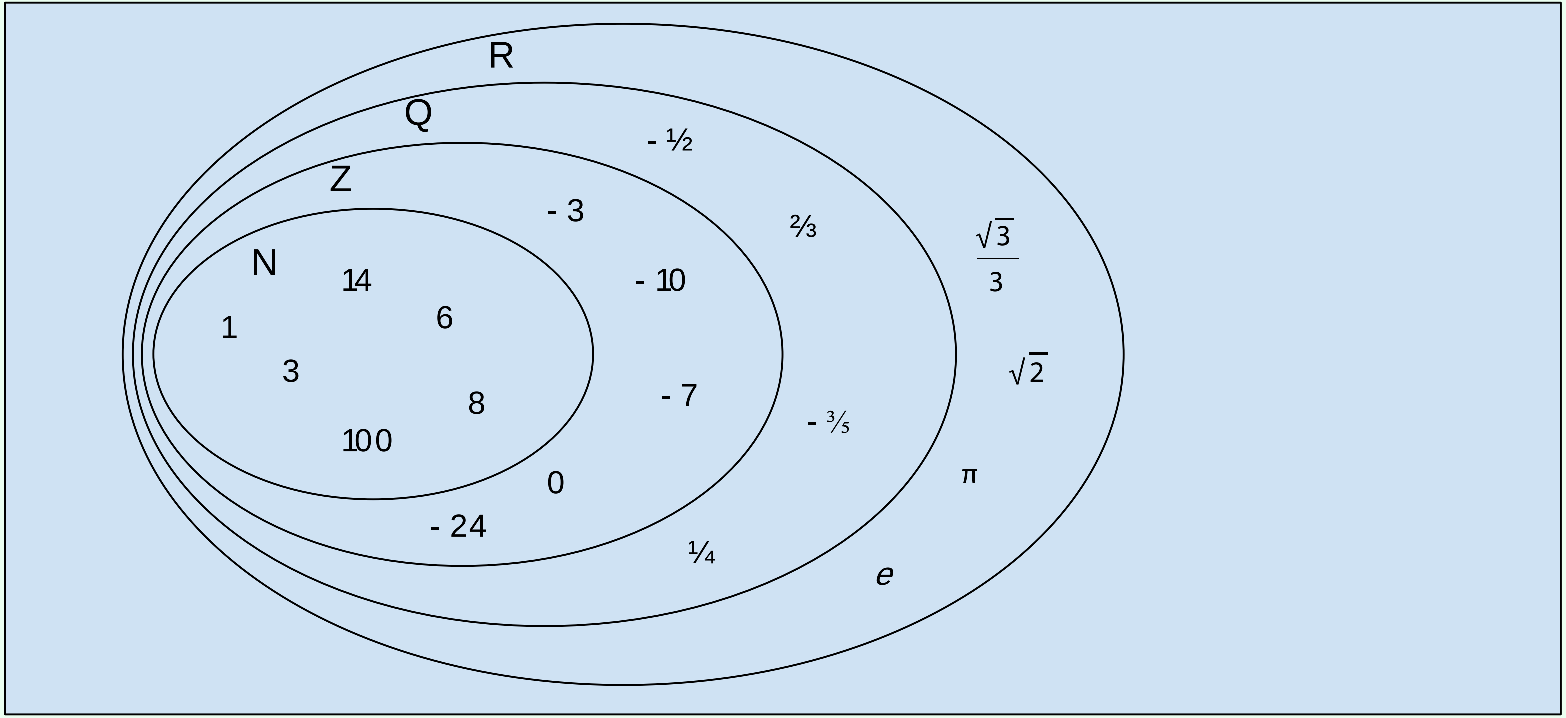
Numbers

❖ Natural numbers	N	{1, 2, 3, 4, 5, 6, …}
❖ Integers	Z	{ .., -3, -2, -1, 0, 1, 2, 3, …}
❖ Rational numbers	Q	{ Z, ½, ⅔, ⅗, ... }
❖ Real numbers	R	{ Q, e, π, √2, …}

What do you think Z^+ represent?

Numbers

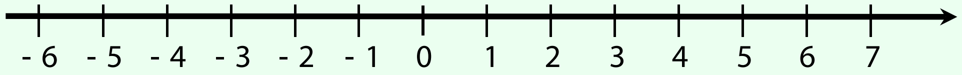
- Natural numbers N $\{1, 2, 3, 4, 5, 6, \ldots\}$
- Integers Z $\{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational numbers Q $\{ Z, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \ldots \}$
- Real numbers R $\{ Q, e, \pi, \sqrt{2}, \ldots\}$



Simple Number Operations

❖ Additions	'plus'	+	2 + 3 = 5
❖ Subtraction	'minus'	-	7 - 6 = 1
❖ Multiplication	'times'	x	4 x 3 = 12
❖ Division	'divide'	÷	18 ÷ 2 = 9

Number Line



The further right the number is, the bigger it is.

Conversely, the further left the number is, the smaller it is.

Simple Operations involving Negative Numbers

- Additions

  $5 + (-3) = 2$
- Subtraction

  $4 - (-1) = 5$
- Multiplication

  $3 \times (-5) = -15$
- Division

  $14 \div (-2) = -7$

(negative) x *(positive)* = *negative*
(positive) x *(negative)* = *negative*
(negative) x *(negative)* = *positive*

(negative) ÷ *(positive)* = *negative*
(positive) ÷ *(negative)* = *negative*
(negative) ÷ *(negative)* = *positive*

Key Words

- Natural Number
- Integer
- Rational number
- Real number
- Addition (plus)
- Subtraction (minus)
- Multiplication (times)
- Division (divide)

- Positive
- Negative
- Number line

The End

Thank you

NUMBERS & OPERATIONS 01-02

Furen International School
O Level Foundation Mathematics

Quick Review 01-01

- Numbers
- Simple number operations
- Number Line
- Simple operations involving negative numbers

Index Notation

- Multiplication of the same number / item can be written using the index notation.
- 3 x 3 can be written as 3^2 (read: 'three squared')
- Similarly, 3 x 3 x 3 can be written as 3^3 (read: 'three cubed')
- So, 3 x 3 x 3 x 3 can be written as 3^4 (read: 'three to the **power** of four'). 4 is called in the **index** (plural: **indices**) and 3^4 is called the **index notation** of 3 x 3 x 3 x 3.

Can you write 5 x 5 x 5 x 5 x 5 x 5 x 5 in index notation?

Surds Notation

- The reverse of the index notation is the surds notation ($\sqrt{\ }$)

  $2^2 = 4 \quad \Leftrightarrow \quad \sqrt{4} = 2$ (read: 'square root of four is two')

  $3^3 = 27 \quad \Leftrightarrow \quad \sqrt[3]{27} = 3$ (read: 'cube root of twenty seven is three')

- We also notice that $4^2 = 16$ and $(-4)^2 = 16$

  Note that the surds notation ($\sqrt{\ }$) only denote the positive square root. To denote both the positive and negative square roots, we use the '**plus minus**' sign ($\pm$)

$$\pm\sqrt{16} = \begin{cases} +\sqrt{16} = 4 \\ -\sqrt{16} = -4 \end{cases}$$

Order of Operations

- If there are multiple operations in an expression, the operations are to be done in the following order:
  - **P**arentheses / Brackets
  - **E**xponent / Index & Roots
  - **M**ultiplication & Division
  - **A**ddition & Subtraction

If there are multiple multiplication / division in an expression, the operations are done from the leftmost to the rightmost

Similarly, if there are multiple addition / subtraction in an expression, the operations are done from the leftmost to the rightmost

Order of Operations

Example:

Evaluate $(4 + 7)^2 \times 3 + 5 - (26 - 8) \div 6$

Solution:

$$
\begin{aligned}
&(4 + 7)^2 \times 3 + 5 - (26 - 8) \div 6 \\
&\quad = (11)^2 \times 3 + 5 - (18) \div 6 \\
&\quad = 121 \times 3 + 5 - 18 \div 6 \\
&\quad = 363 + 5 - 18 \div 6 \\
&\quad = 363 + 5 - 3 \\
&\quad = 368 - 3 \\
&\quad = 365
\end{aligned}
$$

Key Words

- Index (Indices)
- Surd
- Square
- Square root
- Cube
- Cube root
- Parentheses/Bracket
- Exponent

- Evaluate
- Solution
- Leftmost
- Rightmost

The End

Thank you

NUMBERS & OPERATIONS 01-03

Furen International School
O Level Foundation Mathematics

Quick Review 01-02

- Index Notation
- Surd Notation
- Order of Operations

Factors

- The factors of a number is all the natural numbers that divide the aforementioned number exactly, without leaving remainder. W e say that a number is divisible by its factors.

$20 \div 1 = 20 \quad \Rightarrow$ 1 is a factor of 20

$20 \div 2 = 10 \quad \Rightarrow$ 2 is a factor of 20

$20 \div 3 = 6\frac{2}{3} \quad \Rightarrow$ 3 is **not** a factor of 20

$20 \div 4 = 5 \quad \Rightarrow$ 4 is a factor of 20

$20 \div 5 = 4 \quad \Rightarrow$ 5 is a factor of 20

$20 \div 6 = 3\frac{1}{3} \quad \Rightarrow$ 6 is **not** a factor of 20

$\vdots$

Can you find the rest of the factors of 20?

Prime Numbers

- ***Prime numbers*** are numbers who has **exactly two different factors**, 1 and itself.
- ***Composite numbers*** are numbers who has **more than two different factors.**

Example:

a. 3 has exactly two factors, 1 and 3. So 3 is a prime number.
b. The factors of 6 are: 1, 2, 3, 6. So 6 is a composite number.

Prime Numbers

- Prime numbers are numbers who has **exactly two different factors**, 1 and itself.
- Composite numbers are numbers who has **more than two different factors**.

Exercise

1) State if the following numbers are prime numbers or composite numbers.

a)	4	d)	51
b)	7	e)	123
c)	29	f)	149

2) Is there a natural number which is neither prime nor composite? Explain.

Prime Factorisation

- Consider again the number 20. We can express 20 as a product of its **prime factors**.

$$20 = 2 \times 2 \times 5$$
$$= 2^2 \times 5$$

Prime factors are factors which are prime numbers.

- We call 2 and 5 as the prime factors of 20, and the process of expressing 20 as a product of its prime factors is called **prime factorisation** of 20.

Prime Factorisation

Exercise

Find the prime factorisation of the following numbers.

1) 4
2) 18
3) 156
4) 270
5) 441
6) 629

Square Roots and Cube Roots

- Squares of whole numbers (e.g. 1, 4, 9, 16, …) are called **perfect squares**. Similarly, cubes of whole numbers (e.g. 1, 8, 27, 64, …) are called **perfect cubes**.
- We can use prime factorisation to find if the numbers are perfect squares or perfect cubes, and, subsequently, use it to find the square root / cube root of a number.

Square Roots and Cube Roots

Example:

The prime factorisation of 144 is

$$144 = 2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } 3$$
$$= (2 \text{ x } 2 \text{ x } 3) \text{ x } (2 \text{ x } 2 \text{ x } 3)$$
$$= (2 \text{ x } 2 \text{ x } 3)^2$$

∴ 144 is a perfect square.

$$\sqrt{144} = \sqrt{(2 \times 2 \times 3)^2} = 2 \times 2 \times 3 = 12$$

Square Roots and Cube Roots

Example:

The prime factorisation of 1000 is

$$1000 = 2 \text{ x } 2 \text{ x } 2 \text{ x } 5 \text{ x } 5 \text{ x } 5$$
$$= (2 \text{ x } 5) \text{ x } (2 \text{ x } 5) \text{ x } (2 \text{ x } 5)$$
$$= (2 \text{ x } 5)^3$$

∴ 1000 is a perfect cube.

$$\sqrt[3]{1000} = \sqrt[3]{(2 \times 5)^3} = 2 \times 5 = 10$$

Square Roots and Cube Roots

- ❖ If the number in question are not a perfect square / perfect cube, we can't use prime factorisation to find the square root / cube root. In this case, we can find a mental estimation of square roots and cube roots.

<u>Example</u>:

The prime factorisation of 80 is

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$\therefore$ 80 is not a perfect square.

However, we observe that 80 is close to 81 which is a perfect square. Thus,

$$\sqrt{80} \approx \sqrt{81} = 9$$

Square Roots and Cube Roots

Example:

The prime factorisation of 345 is

$$345 = 3 \times 5 \times 23$$

$\therefore$ 345 is not a perfect cube.

However, we observe that 345 is close to 343 which is a perfect cube. Thus,

$$\sqrt[3]{345} \approx \sqrt[3]{343} = 7$$

Square Roots and Cube Roots

❖ The square root and cube root of a number can be found using a calculator.

<u>Exercise</u>

Do a mental estimation for each of the following expressions. Afterwards, use calculator to evaluate the expressions, giving your answer correct to 4 decimal places. Compare the results you get from using both methods.

1) $\sqrt{63}$

2) $\sqrt[3]{124}$

3) $\sqrt{198} - \sqrt[3]{515}$

4) $\frac{2+\sqrt{17}}{\sqrt[3]{28}}$

Determining Prime with Trial Division

- **Trial division** is a method to determine whether a number is a prime by dividing the number by all prime number less than or equal to the square root of itself. This method is useful to determine whether a large number is a prime.

<u>Example</u>:

Is 1993 a prime number?

Since $\sqrt{1993} \approx 44.6$, the largest prime less than or equal to $\sqrt{1993}$ is 43. Dividing 1993 with all prime numbers less than or equal to 43 (2, 3, 5, 7, … , 43), we found out that 1993 is not divisible by any of those primes. Therefore, 1993 is a prime number.

Key Words

- Factor
- Remainder
- Divisible
- Prime number
- Composite number
- Prime factorisation
- Prime factors
- Decimal places
- Calculator
- Perfect square
- Perfect cube

The End

Thank you

NUMBERS & OPERATIONS 01-04

Furen International School
O Level Foundation Mathematics

Quick Review 01-03

- ❖ Factors
- ❖ Prime factorisation
- ❖ Perfect square/Square root
- ❖ Perfect cube/ Cube root

Highest Common Factor (HCF)

❖ Common factors are shared factors between two (or more) numbers.

Example:

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 32 are 1, 2, 4, 8, 16, 32.

The common factors of 24 and 32 are 1, 2, 4, and 8.
∴ The highest common factor of 24 and 32 is 8.

This method is called the **listing method**.

Highest Common Factor (HCF)

- ❖ However, the listing method is not efficient when we are finding out the common factors of large numbers.
- ❖ More efficiently, we can find the highest common factor by looking at the common prime factors of the numbers.

<u>Example</u>:

The prime factorisation of 24 are 2 x 2 x 2 x 3
The prime factorisation of 32 are 2 x 2 x 2 x 2 x 2

The highest common factor of 24 and 32 is 2 x 2 x 2.
∴ The highest common factor of 24 and 32 is 8.

Highest Common Factor (HCF)

Example:

Find the HCF of 432 and 600.

For common prime factors, choose the one with **lower index.**

Solution 1:

$$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$$
$$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2$$

HCF of 432 and 600 is 2 x 2 x 2 x 3 = 24 or 2^3 x 3 =24

Highest Common Factor (HCF)

Example:

Find the HCF of 432 and 600.

Solution 2:

2	432	600	→	divide 432 and 600 by 2 to get 216 and 300
2	216	300	→	divide 216 and 300 by 2 to get 108 and 150
2	108	150	→	divide 108 and 150 by 2 to get 54 and 75
3	54	75	→	divide 54 and 75 by 3 to get 18 and 25
	18	25	→	stop division as 18 and 25 has no common prime factors

HCF of 432 and 600 is 2 x 2 x 2 x 3 = 24

Lowest Common Multiple (LCM)

- ❖ Common multiples are shared multiples between two (or more) numbers.

Example:

The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, ...
The multiples of 18 are 18, 36, 54, 72, 90, ...

The common multiples of 12 and 18 are 36, 72, ...
∴ The lowest common multiple of 12 and 18 is 36.

This method is called the **listing method**.

Lowest Common Multiple (LCM)

- ❖ However, the listing method is tedious when we are writing out the common multiples of large numbers.
- ❖ More efficiently, we can find the lowest common multiple by looking at the prime factorisation of the numbers.

Lowest Common Multiple (LCM)

Observation:

The prime factorisation of 12 are 2 x 2 x 3.
The prime factorisation of 18 are 2 x 3 x 3.

The lowest common multiple of 12 and 18, as found out previously, is
36 = 2 x 2 x 3 x 3.

We can observe that the common prime factors of 12 and 18 (a ‘2’ and a ‘3’) is only counted only once in the prime factorisation of the LCM. Therefore, for larger number, we can find the LCM by using the following method.

Lowest Common Multiple (LCM)

Step 1: Write down the prime factorisation of each number.
Step 2: Identify the common prime factors.
Step 3: The LCM of the numbers are given by the product of the common prime factors and all the other prime factors.

Example:

By listing method, the LCM of 6 and 8 is 24.

$$6 = \boxed{2} \times \boxed{3}$$
$$8 = \boxed{2} \times \boxed{2 \times 2}$$

$\therefore$ The LCM of 6 and 8 is $\boxed{2} \times \boxed{2 \times 2} \times \boxed{3} = 24$.

Lowest Common Multiple (LCM)

Example:

Find the LCM of 10 and 12.

Solution 1:

$$10 = \boxed{2} \times 5$$
$$12 = \boxed{2} \times 2 \times 3$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

LCM of 10 and 12 = 2 x 2 x 5 x 3 = 60

or

$$10 = 2 \times 5$$
$$12 = \boxed{2^2} \times 3$$

For common prime factors, choose the one with **higher index.**

LCM of 10 and 12 is $2^2 \times 3 \times 5 = 60$

Lowest Common Multiple (LCM)

Example:

Find the LCM of 10 and 12.

Solution 2:

$$\begin{array}{r|cc} 2 & 10 & 12 \\ \hline & 5 & 6 \end{array}$$

→ divide 10 and 12 by 2 to get 5 and 6

→ stop division as 5 and 6 has no common prime factors

LCM of 10 and 12 is 2 x 5 x 6 = 60

Application of HCF and LCM

Example:

Elliott buys a new shirt every 9 months and new shoes every 15 months. He bought a new shirt and new shoes together in January 2018. Which month and year will it be when he buys a new shirt and shoes together again?

Solution:

$$9 = 3 \times 3 \qquad\qquad 15 = 3 \times 5$$

$\therefore$ The LCM of 9 and 15 is $3 \times 3 \times 5 = 45$.

So Elliott will buy a new shirt and shoes together again 45 months (3 years, 9 months) after January 2018, which would be October 2021.

Key Words

- HCF (Highest Common Factor)
- LCM (Lowest Common Multiple)
- Lower index
- Higher index

Chapter 01 Key Words

- Natural Number
- Integer
- Rational number
- Real number
- Addition (plus)
- Subtraction (minus)
- Multiplication (times)
- Division (divide)
- Positive
- Negative
- Number line
- Decimal places
- Calculator
- Perfect square
- Perfect cube
- Index (Indices)
- Surd
- Square
- Square root
- Cube
- Cube root

Chapter 01 Key Words

- Evaluate
- Solution
- Leftmost
- Rightmost
- Factor
- Remainder
- Divisible
- Prime number
- Composite number
- Prime factorisation
- Prime factors
- Decimal places
- Calculator
- Perfect square
- Perfect cube
- Parentheses/Bracket
- Exponent
- HCF (Highest Common Factor)
- LCM (Lowest Common Multiple)
- Lower index
- Higher index

The End of Chapter 01

Thank you